# *Proposals*

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***1***. *Delete* $100$ *digits from the number*

$$12345678910111213141516\cdots5960$$

*in such a way as to make the resulting number as small as possible. Repeat the procedure to obtain the resulting number as large as possible.*

***2***. *What is the minimum number of positive integers less than or equal to* $1000$ *that can be chosen to assure that there are two of them which do not have any common prime divisor?*

***3***. *Let* $a, b, c$ *be the length of the sides of a triangle* $ABC$*. Prove that*

$$\sqrt{a}+\sqrt{b}+\sqrt{c} \geq \sqrt{a+b-c}+\sqrt{b+c-a}+\sqrt{c+a-b}$$

***4***. *Let* $m_a, m_b, m_c$ *be the medians of a triangle* $ABC$ *and* $a, b, c$ *be the lengths of its sides. Prove that*

$$\frac{a^2+b^2+c^2}{m_a^2+m_b^2+m_c^2}$$

*does not depend of the triangle* $ABC$ *and determine its value.*

***5***. *Let* $n$ *be a positive integer. Show that*

$$\left(n^3-n\right)\left(5^{8n+4}+3^{4n+2}\right)$$

*is a multiple of* $1902$.

***6***. *Let* $a, b$*, and* $c$ *be positive real numbers. Prove that*

$$1+\frac{3}{ab+bc+ca} \geq \frac{6}{a+b+c}$$

***7***. *Let* $n > 11$ *be a positive integer. Show that* $n^2-19n+89$ *cannot be a perfect square.*

***8***. *Determine all functions* $f : \mathbb{R} \to \mathbb{R}$ *that verify the condition*

$$f(x^2+y) = f(x)+f(y^2),$$

*for all* $x, y \in \mathbb{R}$.

**9**. *Color the vertices of a convex $2n$-gon alternatively with two colors. Determine the number of diagonals with differently colored endpoints.*

**10**. *Find all triplets $(x, y, z)$ of real numbers such that*

$$\left.\begin{array}{l} x^2(y+z)^2 = (5x^2+x+1)y^2z^2 \\ y^2(z+x)^2 = (7y^2+y+1)z^2x^2 \\ z^2(x+y)^2 = (8z^2+z+1)x^2y^2 \end{array}\right\}$$

**11**. *Consider five segments in the plane such that any three of them form a triangle. Prove that at least one of these triangles is acute.*

**12**. *In triangle $ABC$, we draw a perpendicular through the midpoint $D$ of $BC$ to the bisector of angle $A$. Show that this line cuts off segments on the sides $AB$, $AC$ equal respectively to $\frac{1}{2}(AB+AC)$ and $\frac{1}{2}(AB-AC)$.*

**13**. *Let $\{a_n\}_{n\geq 1}$ be the sequence defined by*

$$a_1 = 1, a_2 = 2 \quad \text{and} \quad a_{n+2} = 2a_{n+1} - a_n + 2,\ n \geq 1.$$

*Prove that for any $m$, $a_m a_{m+1}$ is also a term in the sequence.*

**14**. *Find all nonnegative solutions of the equation*

$$\frac{10}{x+10} + \frac{10\cdot 9}{(x+10)(x+9)} + \ldots + \frac{10\cdot 9\cdots 3\cdot 2\cdot 1}{(x+10)(x+9)\cdots(x+2)(x+1)} = 5$$

**15**. *Let $a, b, c$ be any three integers. Prove that*

$$abc(a^3-b^3)(b^3-c^3)(c^3-a^3)$$

*is divisible by $7$.*

**16**. *Let $ABCD$ be a convex quadrilateral and $P, Q$ are the midpoints of $CD$ and $AB$, respectively. Let $AP, DQ$ meet at $X$ and $BP, CQ$ meet at $Y$. Prove that $[ADX] + [BCY] = [PXQY]$.*

**17**. *Consider any convex region in the plane crossed by $\ell$ lines with $p$ interior points of intersection. Find and expression that relates $\ell, p$ and the number $r$ of disjoint subregions created.*

**18**. *Let $x, y, z$ and $n > 1$ be positive integers such that $x^n + y^n = z^n$. Prove that $x, y, z$ are all greater than $n$.*